MTH 2310, FALL 2011

TEST 2 REVIEW

- The test will take the full period.
- You can use a calculator, but you will not need one.
- The test will cover sections 2.1-2.4, 3.1, 3.2, 4.1 and 4.2.
- To study for the test, I recommend looking over your notes and trying to rework old problems from class, HW problems, and questions from previous quizzes. You can also work out problems from the Supplementary Exercises at the end of Chapter 2, 3 and 4. In particular:
  (i) Chapter 2 Supplementary: # 1, 2, 3, 5, 9, 10
  (ii) Chapter 3 Supplementary: # 1, 2, 3, 5 and 6
  (iii) Chapter 4 Supplementary: # 1 (a, b, c, h, i, j) and # 4
  The answers to most of those are in the back of the textbook if you want to check your work.
- As with the quizzes, it is important that you know not just the answer to a question, but also how to explain your answer.

Some problems to work on in class today (most of these are even-numbered problems from the textbook):

(1) True/False: If True, justify your answer with a brief explanation. If False, give a counterexample or a brief explanation.
  (a) If A and B are 3×3 and $B = \begin{bmatrix} \mathbf{b}_1 & \mathbf{b}_2 & \mathbf{b}_3 \end{bmatrix}$, then $AB = \begin{bmatrix} A\mathbf{b}_1 + A\mathbf{b}_2 + A\mathbf{b}_3 \end{bmatrix}$.
  (b) If A is invertible, then the inverse of A^{-1} is A itself.
  (c) Let A be a square matrix. If the equation $A\mathbf{x} = \mathbf{b}$ has at least one solution for each $\mathbf{b}$ in $\mathbb{R}^n$, then the solution is unique for each $\mathbf{b}$.
  (d) If A_1, A_2, B_1 and B_2 are $n \times n$ matrices, $A = \begin{bmatrix} A_1 \\ A_2 \end{bmatrix}$ and $B = \begin{bmatrix} B_1 & B_2 \end{bmatrix}$, then the product BA is defined but AB is not.
  (e) The determinant of a triangular matrix is the sum of the entries on the diagonal.
  (f) If $\det A = 0$, then two rows or two columns are the same, or a row or column is zero.
  (g) A vector space is also a subspace.
  (h) The column space Col A is not affected by elementary row operations on A.

(2) Show that if the columns of B are linearly independent, then so are the columns of AB. (Hint: First, explain why the columns of B being linearly dependent means the same thing as saying that there is a nonzero vector $\mathbf{v}$ such that $B\mathbf{v} = \mathbf{0}$. Now use this to answer the question.)

(3) Find the inverse of $\begin{bmatrix} 1 & -2 & 1 \\ 4 & -7 & 3 \\ -2 & 6 & -4 \end{bmatrix}$.

(4) If L is an $n \times n$ matrix and the equation $L\mathbf{x} = \mathbf{0}$ has the trivial solution, do the columns of L span $\mathbb{R}^n$? Why or why not?

(5) Let $A = \begin{bmatrix} B & 0 \\ 0 & C \end{bmatrix}$, where B and C are square. Prove that if A is invertible then B and C must be invertible. (Note: you cannot just invoke the Invertible matrix Theorem, you have to use block matrix multiplication at some point.)

(6) Calculate $\det \begin{bmatrix} 6 & 3 & 2 & 4 & 0 \\ 9 & 0 & -4 & 1 & 0 \\ 8 & -5 & 6 & 7 & 1 \\ 3 & 0 & 0 & 0 & 0 \\ 4 & 2 & 3 & 2 & 0 \end{bmatrix}$.

(7) Use row reduction to find $\det \begin{bmatrix} 1 & 3 & 3 & -4 \\ 0 & 1 & 2 & -5 \\ 2 & 5 & 4 & -3 \\ -3 & -7 & -5 & 2 \end{bmatrix}$.

(8) Let U be a square matrix such that $U^T U = I$. Show that either $\det U = 1$ or $\det U = -1$.

(9) Let W be the set of all vectors of the form $\begin{bmatrix} 4a+3b \\ 0 \\ a+b+c \\ c-2a \end{bmatrix}$. Is W a subspace of $\mathbb{R}^4$? Explain your reasoning.

(10) Consider the following two systems of equations:

$$\begin{aligned} 5x_1 + x_2 - 3x_3 &= 0 \\ -9x_1 + 2x_2 + 5x_3 &= 1 \\ 4x_1 + x_2 - 6x_3 &= 9 \end{aligned} \quad \text{and} \quad \begin{aligned} 5x_1 + x_2 - 3x_3 &= 0 \\ -9x_1 + 2x_2 + 5x_3 &= 5 \\ 4x_1 + x_2 - 6x_3 &= 45 \end{aligned}$$

It can be shown that the first system has a solution. Use this fact to explain why the second system also has a solution without making any row operations.